Book Review

H. Ratschek and J. Rokne, *New Computer Methods for Global Optimization*, Ellis Horwood Limited, Chichester, Halstead Press (a Div. of Wiley & Sons), New York (1988), iv + 230 pp. $63.96.

This monograph by Helmut Ratschek and Jon Rokne is a brilliant, monumental achievement. It is superb in concept, content, organization and clarity. This is not to say that it will be easy going for the novice, uninitiated in matters of global optimization via interval methods. But it will be well worth the effort, aided as much as possible by the excellent style of the writing. Even those well-versed in the literature of interval analysis may find Chapter Four (Unconstrained Optimization over Unbounded Domains) a bit rough going, not for lack of skill in the presentation by any means, but just because it really is new material and requires stretching the mind into new domains – unbounded domains. This reviewer urges those seriously interested in optimization to take some time to carefully study this work.

There are five chapters. Chapter 1, Some Principles of Optimization Theory, discusses the general optimization problem and some of the classical optimization techniques. Later chapters show how to combine some of these procedures with interval methods.

Chapter 2, Principles of Interval Analysis, motivates and explains the basic techniques on interval (set-valued) computation and theory.

Chapter 3, Global Unconstrained Optimization, introduces the basic interval algorithm, provides convergence conditions, termination criteria, accelerating devices, and numerical examples.

Chapter 4, Unconstrained Optimization over Unbounded Domains, is perhaps the most remarkable chapter in this totally remarkable book. It shows how to implement, on computers, algorithms for minimization over unbounded domains. Prior familiarity with the topological concept of compactification will be an aid to the reader.

Chapter 5, Constrained Optimization, admits at last the full complexity of optimization problems we must deal with.

There are mathematical proofs and numerical examples throughout the work. Algorithms are presented in a clear form which can be directly implemented in programming in any language providing for interval data types (such as Pascal-SC, ACRITH-XSC, etc.). In fact, the author of this review has programmed a version of "algorithm 2" (on pages 174–176 of the book under review) from scratch (including the required interval arithmetic operations) in MS-BASIC for the MacIntosh computer and has successfully solved non-differentiable, nonlinear

constrained minimization problems which have several continuua of local and global minimizers.

The computation produces lists of small boxes whose union contains all the global minimizers. A list of small overlapping boxes can contain an arc of a curve, a section of a surface, etc., perhaps all of whose points are global minimizers.

In favor of clarity for the first book on this subject, the authors have wisely chosen to keep the algorithms as simple as possible. More sophisticated algorithms can greatly improve efficiency. Forthcoming works, especially a book in progress by Eldon Hansen, will show the way to fasten computations of the kind described in the book under review.

Only a few minor typographical errors (or apparent inconsistencies) were noticed by this reviewer. On page 136, line 6 from the bottom: R should have a line over it. Starting at page 140 notation such as f^+, etc. suddenly replaces what had been f^*, etc. for no obvious reason. On page 167, at step 8 in the algorithm, the unequal sign is obviously meant to be an equality sign. As books go, this one is remarkably free of typos. Furthermore, the publisher is to be congratulated for the uncramped and readable size and appearance of the print. It is a welcome treat for these tired old eyes.

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